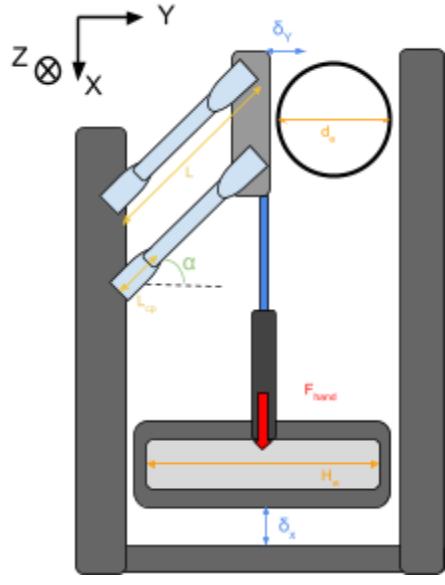


## Mechanical Problem Set #3 - Egg Gripper

**Executive Summary:** Using the principles of Pseudo-Rigid Body Modeling, we were able to obtain mathematical models to guide the design of our final flexure, which starts at a 45 degree angle and displaces a maximum of 8 mm when its handle is pulled, generating 1.68 N of egg normal force and 16 MPa of stress within the flexures. After finding dimensions for a physical model that ensured the flexures and egg did not break while gripping the egg, we fabricated our gripper and tested to validate functional requirements (FRs).

### Math Model:



Our mathematical model is based on pseudo-rigid body modeling of a 4-bar linkage. We established a base geometry and identified key dimensions for us to reference in our kinematic and stress equations. Our flexures are cut such that, in their unstrained configuration, they are at a starting angle  $\alpha$  from perpendicular to the fixed wall. The angle of deflection that the flexures have rotated about the modeled hinge is represented by  $\phi$ . Each of these hinges are modeled to be a distance  $l_{cp}$  from the fixed walls. They are also modeled as torsional springs, which absorb more energy as the angle  $\phi$  increases. From this, we were able to derive the key kinematic equations of:

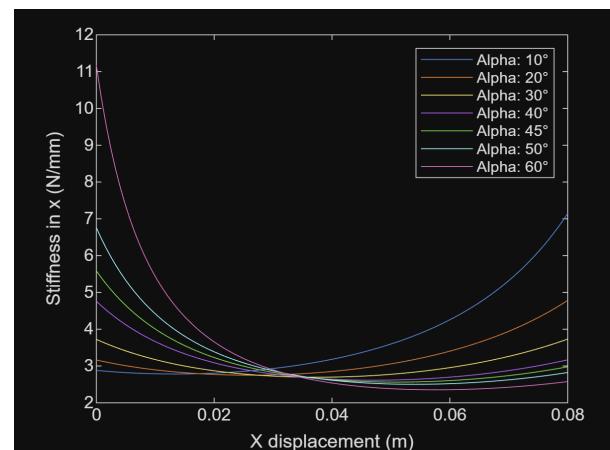
$$\delta_x = l * \sin(\alpha) - [2l_{cp} \sin(\alpha) + \gamma * l * \sin(\alpha - \phi)]$$

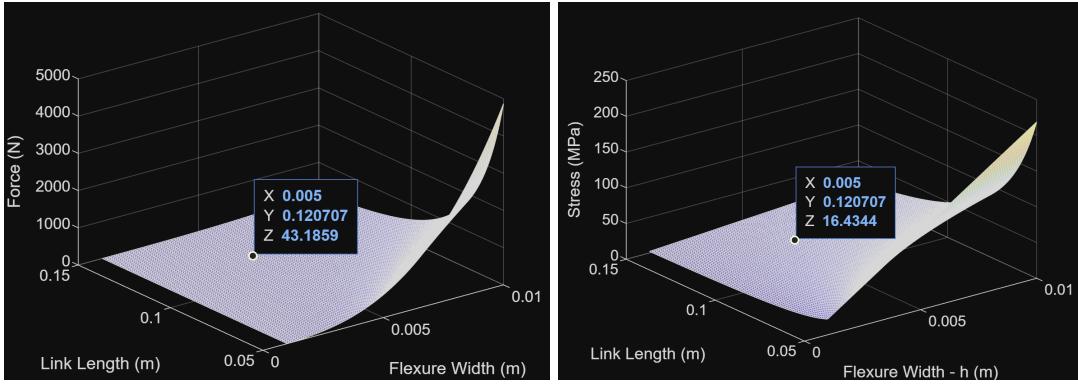
$$\delta_y = 2l_{cp} * \cos(\alpha) + \gamma * l * \cos(\alpha - \phi) - l * \cos(\alpha)$$

Subsequently, we derived an energy equation for our fixed-guided 4-bar mechanism:

$$U(x) = 2K_t(\alpha - \sin^{-1}(\sin(\alpha) - x/(\gamma * l)))^2$$

We differentiated to find the force in terms of x displacement,  $F(x)$ . Differentiating again, we found the stiffness in x,  $K_x(x)$ . The following plots show  $K_x(x)$  over a range of the first 80mm and 10mm, respectively from left to right. We sought to find an angle  $\alpha$  that would retain a generally constant stiffness as it reached our target x-displacement of 8mm.



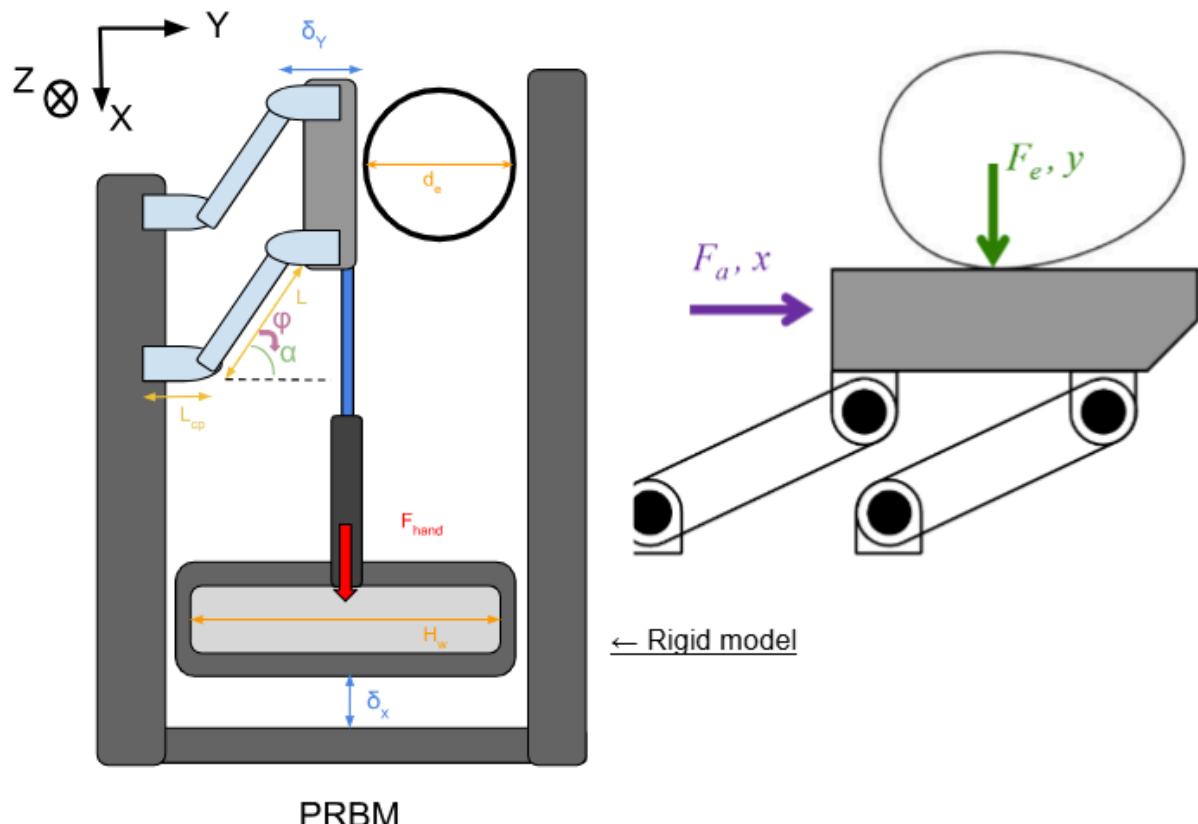


We then proceeded to optimize our flexure geometry by generating surface plots of different combinations of angles and parameters. We used these plots to find suitable values of flexure width, length, and starting angle to allow for the required 8 mm of travel, while not requiring a prohibitive amount of force to obtain that travel and prevent yielding. We found a combination that required only 43 N of actuation force, a flexure length of 121mm, and a flexure width of 5 mm. We determined the stress that would be imparted onto the flexures by this force and displacement. We calculated the stress using the following equation, and found that we were satisfying our FR for a safety factor of 2, where  $c$  is half the flexure width.  $\sigma_{MAX} = \frac{K_t * \phi * c}{2 * I}$

**Verification Method & Results:** To verify the transmission ratio and the force applied to the egg, we put an egg in the tool clamp and released until the egg dropped. Using the relation:  $\mu_s * F_{e,y} = m * g$  and measuring the force from the actuator, we were able to determine the force being applied to the egg before it dropped in order to hold it up. See Appendix I for functional requirements and validation results. Also see Appendix II for FEA.

**Discussion/Learnings:** After creating a prototype using laser-cut acrylic, we found that two contact points from the gripper allowed for free rotation of the egg about the axis connecting those two contact points, leading to some instability. To solve this issue, we referred to the sphere in a v-groove kinematic coupling from earlier in the course. The combination of the nesting force from the gripper onto the v-groove constrained this rotation, providing better stability. A fabrication process we learned throughout this project was the importance of calibrating the waterjet. With zero offset, our values for the measured dimensions of a small rectangular piece were smaller from CAD by an average of 0.86 mm. Half of this value, 0.43 mm, was used as the kerf offset for the waterjet, leading to dimensions with an accuracy of  $\pm 0.01$ mm.

## Appendix I



Variable Name	Symbol	Range	Validation	Justification
Egg Friction Force	$F_f$	0.52 - 0.73 N	0.58 N	Lower bound based on the average egg mass range. Friction force = $m_{\text{egg}} \cdot g$ <a href="https://www.egginfo.co.uk/egg-facts-and-figures/industry-information/egg-size-s">https://www.egginfo.co.uk/egg-facts-and-figures/industry-information/egg-size-s</a>
Egg Normal Force	$F_{e,y}$	$1.6 \pm 0.4$ N	$0.58 \text{ N} / 0.35 = 1.68 \text{ N}$ The gripper applied this amount of normal force to successfully grip the eggs we tested on.	Upper bound based on a <a href="#">maximum compression force of ~45N</a> The average mass of an egg is 56 g $0.3 < \mu_s < 0.5$ based on the friction of ABS and ceramic
Max Stress on Egg	$\sigma$	$0 \pm 10$		$\mu_s \cdot F_{e,y} > m \cdot g$ $F_{e,y} = 0.59/0.3 = 2 \text{ N}$ $F_{e,y} = 0.59/0.5 = 1.2 \text{ N}$ <a href="https://www.stonybrook.edu">https://www.stonybrook.edu</a>

from 4-bar mechanism		MPa	Divided normal force by contact surface area. $1.68 \text{ N} / (3(3.68*3.89)) = 0.0391 \text{ MPa}$	<a href="u/commcms/wanglf/_pdf/72-ActaBio-2024.pdf">u/commcms/wanglf/_pdf/72-ActaBio-2024.pdf</a> Fracture stress of eggshells is about 19.9 MPa, FR determined using a SF of 2.
Minimum Gripper Closing Distance	$\delta_Y$	8.5 mm	8.00 mm	Large enough for an egg to initially fit between the grippers and to be gripped.  Clearance between the egg and the initial position of the actuator is 3 mm so the egg can fit inside the device plus the distance that compresses the egg with a minimum of 10 N (constant egg size given that we provide the egg)
Actuator Displacement	$\delta_A$	8.5 - 70 mm	9.05 mm	Upper bound defined by <a href="#">ergonomic handle standards for gripping tools</a> .  Gives a maximum distance that the hand will pull on the device to exert the necessary force on the egg  Lower bound determined such that when $\delta_A$ displaces 1 mm, $\delta_Y$ also displaces at least 1 mm (input displacement is not being amplified)
Displacement Ratio	DR	1 - 8.2	1.131	Upper bound: $\delta_A/\delta_Y$ . The lower bound is based on 1:1. Ratio will not be a linear relationship.
Maximum Hand Force at Max Actuation (when $\delta_A = 0$ )	$F_{\text{hand}}$	$40 \pm 10 \text{ N}$	43.3 N	Based on experimentally gripping a force gauge, the max force a person would apply at the hard stop of the gripper is $F_{\text{hand}}$ to grip comfortably with variability in maintaining that grip force.
Force on actuator needed to grip egg	$F_A$	$10 \pm 5 \text{ N}$	12.5 N	Experimentally, a person has a grip force of about 10 N for 20 seconds and is able to retain this resolution within 5 N.
Transmission Ratio	TR	Nominal:	$F_A/F_{e,y} = 12.5 \text{ N}/1.68 \text{ N}$	$F_A/F_{e,y}$ . Uncertainties are

		6.25 $4.17 < \text{TR} < 7.5$	= 7.4	based on the variability in applied hand force and egg force.
Handle Width	$H_w$	$70 \pm 10$ mm	70.06 mm	This is the range of values for the width of Lindsay's four fingers and Michael's four fingers
Max Allowable Stress in Flexures	$\sigma_F$	24 MPa	16 MPa	<a href="#">Thermoplastics - Physical Properties</a> ABS has a yield stress of around 48 MPa; applied SF of 2.
Minimum stiffness in z-direction	$k_z$	5 N/mm	Without backing: $3.6\text{N}/1.25\text{ mm} = 2.88\text{ N/mm}$ With Backing: $7.4\text{ N} / 1.29\text{ mm} = 5.74\text{ N/mm}$	Stiffness required such that the gripper is stable in the direction not intended for motion (z). The gripper flexures must not deflect significantly under the weight of the mechanism.
Mass	$m$	$0.25\text{ lb} < m < 5\text{ lbs}$	0.241 lbs (just flexure) 0.419 lbs with flexure guide and PRBM overlay.	Needs to be able to be held level in one hand. Shouldn't be any more difficult than holding an ipad which weighs 1.5 lbs, but also has a longer moment arm than our device will

## Appendix II

